

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

Cambridge Ordinary Level

## **MARK SCHEME for the October/November 2014 series**

### **4037 ADDITIONAL MATHEMATICS**

**4037/23**

Paper 2, maximum raw mark 80

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1	(i)	$f(2)=0 \rightarrow 3(2)^3+8(2)^2-33(2)+p=0$ correct working to $p = 10$ AG method for quadratic factor $f(x) = (x-2)(3x^2+14x-5)$	M1 A1 M1 A1	factorise or solve quadratic factor = 0
	(ii)	$f(x) = (x-2)(3x-1)(x+5)$ $f(x)=0 \rightarrow x=2, -5, \frac{1}{3}$	M1 A1	
2	(i)	${}^{12}C_4 = 495$	B1	
	(ii)	${}^7C_2 \times {}^5C_2 = 21 \times 10$ $= 210$	M1 A1	
	(iii)	not K and B = ${}^6C_2 \times {}^4C_1 = 15 \times 4 = 60$ K and not B = ${}^6C_1 \times {}^4C_2 = 6 \times 6 = 36$ $60 + 36$ $96$  OR K and B = ${}^6C_1 \times {}^4C_1 = 6 \times 4 = 24$ not K and not B = ${}^6C_2 \times {}^4C_2 = 15 \times 6 = 90$ $210 - 90 - 24$ $96$	B1 B1 M1 A1  B1 B1 M1 A1	
3	(i)	C is (1, 6) D is (1, 6) + (12, 9) $= (13, 15)$	B1 M1 A1ft	correct completion      www
	(ii)	gradient of CD = $\frac{15-6}{13-1} \left( = \frac{3}{4} \right)$  gradient of AB = $\frac{10-2}{-2-4} \left( = \frac{8}{-6} = \frac{-4}{3} \right)$  $\frac{3}{4} \times \frac{-4}{3} = -1$ lines are perpendicular	B1ft  B1  B1	
	(iii)	$\text{area} = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times 10 \times 15$  $= 75$ or array method	M1  A1	

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4	(i) $2000 = 1000e^{a+b} \rightarrow a+b = \ln 2$ (ii) $3297 = 1000e^{2a-b} \rightarrow 2a+b = \ln 3.297$ oe (iii) Solve for one value $a = 0.5$ and $b = 0.193$ or $0.19$ (iv) $n = 10$ $P = 1000e^{5.193}$ $= \$180\,000.$	B1 M1 A1 M1 A1 M1 A1	substitution of 2, 3297 and rearrange
5	(i) $\overline{OX} = \mu(a+b)$ (ii) $\overline{RP} = b - 3a$ or $\overline{RX} = \lambda(b - 3a)$ oe $\overline{OX} = 3a + \lambda(b - 3a)$ (iii) $\overline{OX} = \overline{OX}$ and equate both coefficients $\mu = 3 - 3\lambda$ $\mu = \lambda$ $\mu = \lambda = 0.75$ $\frac{RX}{XP} = 3$ or 3:1	B1 B1 B1 M1 A1 A1ft	$\frac{\lambda}{1-\lambda}$
6	(i) $m = 4$ equation of line is $\frac{\ln y - 39}{3^x - 9} = \frac{39 - 19}{9 - 4}$ $\ln y = 4(3^x) + 3$ (ii) $x = 0.5 \rightarrow \ln y = 4\sqrt{3} + 3 = 9.928$ $y = 20\,500$ (iii) Substitutes $y$ and rearrange for $3^x$ Solve $3^x = 1.150$ $x = 0.127$	B1 M1 A1ft M1 A1 M1 M1 A1	forms equation of line ft only on their gradient correct expression for $\ln y$

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7	(i)	$x = \frac{2}{y} + 1 \rightarrow y = \frac{2}{x-1}$ $f^{-1}(x) = \frac{2}{x-1}$	M1	any valid method
	(ii)	$gf(x) = \left(\frac{2}{x} + 1\right)^2 + 2$	B2/1/0	-1 each error
	(iii)	$fg(x) = \frac{2}{x^2 + 2} + 1$	B2/1/0	-1 each error
	(iv)	$ff(x) = \frac{2}{\frac{2}{x} + 1} + 1 = \frac{2x}{x+2} + 1$ $= \frac{3x+2}{x+2}$ $\frac{3x+2}{x+2} = x \rightarrow x^2 - x - 2 = 0$ $(x-2)(x+1) = 0$ $x = 2 \text{ only}$	M1	correct starting expression
			A1	correct algebra to given answer
			M1	form and solve 3 term quadratic
			A1	
8	(i)	$v = C + K\sin 2t \quad C \neq 0$ $v = 5 + 6\sin 2t$ $a = 12\cos 2t$	M1	
			A1	
			A1ft	
	(ii)	$a = 0 \rightarrow \cos 2t = 0 \text{ and solve}$ $t = \frac{\pi}{4} \text{ or } 0.785 \text{ or } 0.79$ $v = 5 + 6\sin \frac{\pi}{2} = 11$	M1	set $a = 0$ and solve for $t$
			A1	
			A1ft	ft only on $K$
	(iii)	$v = 2 \rightarrow \sin 2t = -\frac{1}{2} \text{ and solve}$ $t = \frac{7\pi}{12} \text{ or } 1.83 - 1.84$ $a = 12\cos \frac{7\pi}{6} = -6\sqrt{3} \text{ or } -10.4$	M1	set $v = 2$ and solve for $t$
			A1	
			A1	

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<p><b>9 (i)</b></p> $\frac{dy}{dx} = 4 - \frac{1}{(x-2)^2}$ $\frac{dy}{dx} = 0 \rightarrow (x-2)^2 = \frac{1}{4}$ $(4x^2 - 16x + 15 = 0)$ <p><math>x = 2.5</math> or <math>1.5</math>  <math>y = 12</math> or <math>4</math></p> $\frac{d^2y}{dx^2} = 2(x-2)^{-3}$ <p><math>x = 2.5 \rightarrow \frac{d^2y}{dx^2} &gt; 0 \rightarrow</math> minimum  <math>x = 1.5 \rightarrow \frac{d^2y}{dx^2} &lt; 0 \rightarrow</math> maximum</p> <p><b>(ii)</b></p> <p><math>x = 3 \rightarrow \frac{dy}{dx} = 3</math></p> <p>Use <math>m_1 m_2 = -1</math> for gradient normal from gradient tangent</p> <p>Eqn of normal : <math>\frac{y-13}{x-3} = -\frac{1}{3}</math></p> <p>Intersection of norm and curve</p> $14 - \frac{x}{3} = 4x + \frac{1}{x-2}$ $13x^2 - 68x + 87 = 0$ $x = \frac{29}{13} \text{ or } 2.23$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b> <b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1ft</b></p> <p><b>M1</b> <b>DM1</b> <b>A1</b></p>	<p>solve 3 term quadratic from <math>\frac{dy}{dx} = 0</math></p> <p><math>x</math> values or 1 pair  <math>y</math> values or 1 pair</p> <p>use <math>\frac{d^2y}{dx^2}</math> with solution from <math>\frac{dy}{dx} = 0</math></p> <p>both identified                      www</p> <p>must use numerical values</p> <p>equation and attempt to simplify  attempt to solve 3 term quadratic</p>
<p><b>10 (i)</b></p> $\text{LHS} = \frac{1 + \cos x + 1 - \cos x}{(1 - \cos x)(1 + \cos x)}$ $= \frac{2}{1 - \cos^2 x}$ $= \frac{2}{\sin^2 x} = \text{RHS}$ <p><b>(ii)</b></p> $2\text{cosec}^2 x = 8$ $\sin^2 x = \frac{1}{4}$ $\sin x = \pm \frac{1}{2}$ $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>correct fraction</p> <p>correct evaluation</p> <p>use of <math>1 - \cos^2 x = \sin^2 x</math> and completion of fully correct proof</p> <p>identity used</p>